Hill & Lowe Educational Services, Inc.
The Hill & Lowe Foundation
Exploratorium Academy

Curriculum Design

Mathematics Education

The following are State Approved Courses and Course Descriptions for (Alabama) High School Students. These and other correlated content standards may be found on our official web-page.

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Approved Courses of Study – Mathematics
State of Alabama

ALGEBRA I

Algebra I is a formal, in-depth study of algebraic concepts and the real number system. In this course students develop a greater understanding of and appreciation for algebraic properties and operations. Algebra I reinforces concepts presented in earlier courses and permits students to explore new, more challenging content which prepares them for further study in mathematics. The course focuses on the useful application of course content and on the development of student understanding of central concepts. Appropriate use of technology allows students opportunities to work to improve concept development. As a result, students are empowered to perform mathematically, both with and without the use of technological tools.

Because of its importance in the development of mathematical empowerment, Algebra I is required for all students. The content is also a central component of formal state-level assessments at the secondary level. To better meet the needs of students of varying abilities, school systems may offer Algebra I (140 hours/one credit) or Algebra IA and IB (280 hours/two credits). If systems choose to offer Algebra I in the eighth grade, the course must include the minimum required content as prescribed in this course of study.

Number and Operations

Students will:

1. Simplify numerical expressions using properties of real numbers and order of operations, including those involving square roots, radical form, or decimal approximations.

   Example: Express $\sqrt{27} + \sqrt{75}$ in simplified form.

   - Applying laws of exponents to simplify expressions, including those containing zero and negative integral exponents

Algebra

2. Analyze linear functions from their equations, slopes, and intercepts.
   - Finding the slope of a line from its equation or by applying the slope formula
   - Determining the equations of linear functions given two points, a point and the slope, tables of values, graphs, or ordered pairs
   - Graphing two-variable linear equations and inequalities on the Cartesian plane
3. Determine characteristics of a relation, including its domain, range, and whether it is a function, when given graphs, tables of values, mappings, or sets of ordered pairs.

- Finding the range of a function when given its domain
  Example: finding the range of $f(x) = -x^2 + 2x - 3$ when given the domain {-4, -2, 0, 2, 4}

4. Represent graphically common relations, including x = constant, y = constant, y = x, $y = \sqrt{x}$, $y = x^2$, and $y = |x|$.
   - Identifying situations that are modeled by common relations, including x = constant, y = constant, y = x, $y = \sqrt{x}$, $y = x^2$, and $y = |x|$

5. Perform operations of addition, subtraction, and multiplication on polynomial expressions.
   - Dividing by a monomial

6. Factor binomials, trinomials, and other polynomials using GCF, difference of squares, perfect square trinomials, and grouping.

7. Solve multistep equations and inequalities including linear, radical, absolute value, and literal equations.
   Examples: solving for x in problems such as $\sqrt{x} - 4 = 0$, $\sqrt{x-4} < 2$, $|x| = 6$, $|x+3| \geq 10$, and $y = mx + b$
   - Writing the solution of an equation or inequality in set notation
     Example: finding the solution of $|x+3| \geq 10$ to be $\{x \mid x \geq 7 \text{ or } x \leq -13\}$
   - Graphing the solution of an equation or inequality
   - Modeling real-world problems by developing and solving equations and inequalities, including those involving direct and inverse variation

8. Solve systems of linear equations and inequalities in two variables graphically or algebraically.
   - Modeling real-world problems by developing and solving systems of linear equations and inequalities

9. Solve quadratic equations using the zero product property.
   - Approximating solutions graphically and numerically

Geometry

10. Calculate length, midpoint, and slope of a line segment when given coordinates of its endpoints on the Cartesian plane.
    - Deriving the distance, midpoint, and slope formulas

Measurement

11. Solve problems algebraically that involve area and perimeter of a polygon, area and circumference of a circle, and volume and surface area of right circular cylinders or right rectangular prisms.
    - Applying formulas to solve word problems
      Example: finding the radius of a circle with area 75 square inches

Data Analysis and Probability

12. Compare various methods of data reporting, including scatterplots, stem-and-leaf plots, histograms, box-and-whisker plots, and line graphs, to make inferences or predictions.
    - Determining effects of linear transformations of data
      Example: The mean score on an algebra test is 78. If the teacher adds five points to each student's grade, the mean score will be 83.
    - Determining effects of outliers
    - Evaluating the appropriateness of the design of a survey

13. Identify characteristics of a data set, including measurement or categorical and univariate or bivariate.
    Example: conducting a survey of 100 students to determine whether boys and girls prefer to watch the same genres of movies to get a bivariate, categorical data set

14. Use a scatterplot and its line of best fit or a specific line graph to determine the relationship existing between two sets of data, including positive, negative, or no relationship.

15. Estimate probabilities given data in lists or graphs.
    - Comparing theoretical and experimental probabilities

GEOMETRY

Geometry provides students with knowledge about shapes and properties and assists with the development of spatial sense, critical for further study in mathematics and for everyday life. Because of its importance in the development of mathematical empowerment, this course is required for all students. To better meet the needs of students of varying abilities, school systems may offer Geometry (140 hours/one credit) or Geometry A and B (280 hours/two credits).

Traditionally, writing proofs has been a major emphasis in Geometry. While in recent years this focus has diminished, Geometry continues to provide an excellent context for developing students' abilities to reason and write proofs. In this course, students are engaged in problematic situations in which they form conjectures, determine the validity of these conjectures, and defend their conclusions to classmates. Emphasis is placed on the power of deductive reasoning, expressed either informally or formally in a variety of formats. The use of technology as a powerful mathematical tool is also encouraged. Technology may be used for exploring geometric situations or may be incorporated into technological applications such as dynamic geometry software to support classroom instruction.

Algebra

Students will:

1. Determine the equation of a line parallel or perpendicular to a second line through a given point.

Geometry

2. Justify theorems related to pairs of angles, including angles formed by parallel and perpendicular lines, vertical angles, adjacent angles, complementary angles, and supplementary angles.
   Example: proving vertical angles congruent

3. Verify the relationships among different classes of polygons by using their properties.
   Example: showing that a square has all the properties of both a rectangle and a rhombus

   - Determining the missing lengths of sides or measures of angles in similar polygons

4. Determine the measure of interior and exterior angles associated with polygons.
   - Verifying the formulas for the measures of interior and exterior angles of polygons inductively and deductively

5. Solve real-life and mathematical problems using properties and theorems related to circles, quadrilaterals, and other geometric shapes.
   Example: finding the center of a solid wooden wheel using the perpendicular bisectors of two chords

   - Determining the equation of a circle given its center and radius

6. Apply the Pythagorean Theorem to solve application problems, expressing answers in simplified radical form or as decimal approximations, using Pythagorean triples when applicable.

7. Use the ratios of the sides of special right triangles to find lengths of missing sides.
   - Deriving the ratios of the sides of 30-60-90 and 45-45-90 triangles

8. Deduce relationships between two triangles, including proving congruence or similarity of the triangles from given information, using the relationships to solve problems and to establish other relationships.
   - Determining the geometric mean to find missing lengths in right triangles

9. Use inductive reasoning to make conjectures and deductive reasoning to justify conclusions.
   - Recognizing the limitations of justifying a conclusion through inductive reasoning

10. Find the missing measures of sides and angles in right triangles by applying the right triangle definitions of sine, cosine, and tangent.

11. Determine the areas and perimeters of regular polygons, including inscribed or circumscribed polygons, given the coordinates of vertices or other characteristics.

12. Apply distance, midpoint, and slope formulas to solve problems and to confirm properties of polygons.
    Examples: finding the area of a rectangle given the coordinates of its vertices, showing that the median of a trapezoid is half the sum of the bases

13. Identify the coordinates of the vertices of the image of a given polygon that is translated, rotated, reflected, or dilated.
    Example: rotating a triangle a given number of degrees around a specific point, comparing the vertices of the image and preimage

14. Classify polyhedrons according to their properties, including the number of faces.
    Example: identifying a polyhedron having 6 vertices and 12 edges

    - Identifying Euclidean solids

Measurement

15. Calculate measures of arcs and sectors of a circle from given information.
    Examples: finding the area of a sector given its arc length and radius, finding the arc length of a sector given its area and radius, finding the area or arc length given the measure of the central angle and the radius

16. Calculate surface areas and volumes of solid figures, including spheres, cones, and pyramids.
    - Developing formulas for surface area and volume of spheres, cones, and pyramids
    - Calculating specific missing dimensions of solid figures from surface area or volume
    - Determining the relationship between the surface areas of similar figures and volumes of similar figures

Data Analysis and Probability

17. Analyze sets of data from geometric contexts to determine what, if any, relationships exist.
    Example: Collect data and create a scatterplot comparing the perimeter and area of various rectangles. Determine whether a line of best fit can be drawn.

    - Distinguishing between conclusions drawn when using deductive and statistical reasoning
    - Calculating probabilities arising in geometric contexts
      Example: finding the probability of hitting a particular ring on a dart board whose rings are formed by equally spaced concentric circles

18. Construct with precision a circle graph to represent data from given tables or classroom experiments.

ALGEBRA II WITH TRIGONOMETRY

Algebra II with Trigonometry focuses on problem-solving skills that use a variety of methods to encourage the development of improved communication skills and foster a deeper understanding of the content area. In order to provide students with an appreciation of the power of algebra, applications involving real-life situations are incorporated throughout the course. The use of appropriate technology is also encouraged.

Algebra II with Trigonometry is required for all students seeking the *Alabama High School Diploma with Advanced Academic Endorsement*. Although this course is valuable for all students, it is strongly recommended for students who intend to pursue postsecondary studies. Prerequisites for the course are Algebra I and Geometry. Credit cannot be awarded for both Algebra II with Trigonometry and Algebra II.

Number and Operations

Students will:

1. Determine the relationships of subsets of complex numbers.
   Example: using Venn diagrams or tree diagrams to show how subsets of complex numbers are related

2. Simplify expressions involving complex numbers, using order of operations and including conjugate and absolute value.
   Examples: simplifying $\sqrt{-8}$, $(4-2i)^2$, and $\frac{3+i}{3-i}$

Algebra

3. Analyze families of functions, including shifts, reflections, and dilations of $y = \frac{k}{x}$ (inverse variation), $y = kx$ (direct variation/linear), $y = [x]$ (greatest integer), $y = x^2$ (quadratic), $y = a^x$ (exponential), and $y = \log_a x$ (logarithmic).
   Example: comparing the graphs of $y = 2^x$, $y = 2^x + 1$, $y = 2^{x+1}$, and $y = -2^x$

   - Identifying the domain and range of a relation given its graph, a table of values, or its equation, including those with restricted domains
     Example: finding the domain of $y = \frac{1}{x-3}$ or $y = \sqrt{x-2}$
   - Identifying real-world situations corresponding to families of functions

4. Determine approximate real zeros of functions graphically and numerically and exact real zeros of polynomial functions.
    - Using the zero product property, completing the square, and the quadratic formula
    - Deriving the quadratic formula

5. Identify the characteristics of quadratic functions from their roots, graphs, or equations.
    - Generating an equation when given its roots or graph
    - Graphing a function when given its equation
        Examples: graphing equations of the form $y = a(x-h)^2 + k$; graphing equations of the form $y = ax^2 + bx + c$
    - Determining the maximum or minimum values of quadratic functions both graphically and algebraically
    - Applying functions to real-world problems

6. Perform operations on functions, including addition, subtraction, multiplication, division, and composition.
    - Determining the inverse of a function or a relation
    - Performing operations on polynomial and rational expressions containing variables
        Example: simplifying $\frac{3}{x+5} + \frac{5}{x^2+6x+5}$
    - Constructing graphs by analyzing their functions as sums, differences, or products

7. Solve equations, inequalities, and applied problems involving absolute values, radicals, and quadratics over the complex numbers, as well as simple trigonometric, exponential, and logarithmic functions.
    Example: solving $x^2 - 8x > -12$, $3^x = 81$, $2\sin^2 x + \sin x = 0$, or $\log_x 2 = 5$

    - Solving equations using laws of exponents, including rational and irrational exponents
    - Expressing the solution of an equation, inequality, or applied problem as a graph on a number line or by using set or interval notation

8. Solve systems of linear equations or inequalities in two or three variables using algebraic techniques, including those involving matrices.
    Example: solving a system of linear equations using augmented matrices and row operations, matrix operations of a graphing calculator, or substitution

    - Evaluating the determinant of a 2x2 or 3x3 matrix
    - Solving word problems involving real-life situations

9. Graph trigonometric functions of the form $y=a\sin(bx)$, $y=a\cos(bx)$, and $y=a\tan(bx)$.

- Determining period and amplitude of sine, cosine, and tangent functions from graphs or basic equations
  Example: solving problems involving harmonic motion
- Determining specific unit circle coordinates associated with special angles

10. Solve general triangles, mathematical problems, and real-world applications using the Law of Sines and the Law of Cosines.
    - Deriving formulas for Law of Sines and Law of Cosines
    - Determining area of oblique triangles

11. Define the six trigonometric functions using ratios of the sides of a right triangle, coordinates on the unit circle, and the reciprocal of other functions.

12. Verify simple trigonometric identities using Pythagorean and/or reciprocal identities.
    Example: verifying $\cos^2 \alpha + \tan^2 \alpha \cos^2 \alpha = 1$

Data Analysis and Probability

13. Use different forms of representation to compare characteristics of data gathered from two populations.
    - Evaluating the appropriateness of the design of an experimental study
    - Describing how sample statistics reflect values of population parameters

14. Determine an equation of linear regression from a set of data.
    - Examining data to determine if a linear, quadratic, or exponential relationship exists and to predict outcomes

15. Calculate probabilities of events using the laws of probability.
    - Using permutations and combinations to calculate probabilities
    - Calculating conditional probability
    - Calculating probabilities of mutually exclusive events, independent events, and dependent events

PRECALCULUS

Precalculus is designed primarily for those students considering careers in mathematical or scientific fields of study. Following the successful completion of Algebra II with Trigonometry, students are prepared for this challenging curriculum that includes an expanded study of polynomial functions, conic sections, logarithmic and exponential equations, and the real-life applications of these topics.

Students are challenged to defend and support their conclusions from problematic situations. Working in both individual and group settings, students apply a variety of problem-solving strategies, incorporating the use of graphing calculators or other technological tools that extend beyond the traditional paper-and-pencil drill and practice.

Number and Operations

Students will:

1. Perform the vector operations of addition, scalar multiplication, and absolute value.
   - Determining coincidence, parallelism, collinearity, or perpendicularity of vectors
   - Using vectors to model real-life and mathematical situations

2. Define e using the limit forms of $\sum_{n=0}^{\infty} \frac{1}{n!}$, $\lim_{n\to\infty} \left(+\frac{1}{n}\right)^{n}$, and $\lim_{n\to 0} \left(+n\right)^{\frac{1}{n}}$.

Algebra

3. Graph conic sections, including parabolas, hyperbolas, ellipses, circles, and degenerate conics, from second-degree equations.
   Example: graphing $x^2 - 6x + y^2 - 12y + 41 = 0$ or $y^2 - 4x + 2y + 5 = 0$

   - Formulating equations of conic sections from their determining characteristics
     Example: writing the equation of an ellipse with center (5, -3), horizontal major axis of length 10, and minor axis of length 4

4. Analyze the graphs of rational, logarithmic, exponential, trigonometric, and piecewise-defined functions by determining the domain and range; identifying any vertical, horizontal, or oblique asymptotes; and classifying the function as

increasing or decreasing, continuous or discontinuous, and noting the type of discontinuity if one exists.

- Approximating rates of change using the difference quotient

5. Analyze the effects of parameter changes on the graphs of trigonometric, logarithmic, and exponential functions.
   Example: explaining the relationship of the graph of $y = e^{x-2}$ to the graph of $y = e^{x}$

   - Determining the amplitude, period, phase shift, domain, and range of trigonometric functions and their inverses

6. Apply the laws of logarithms to simplify expressions and to solve equations using common logarithms, natural logarithms, and logarithms with other bases.

7. Solve trigonometric equations and inequalities using sum, difference, and half- and double-angle identities.
   - Verifying trigonometric identities

8. Use parametric equations to represent real-life and mathematical situations.

9. Solve applied problems involving sequences with recurrence relations.
   - Determining characteristics of arithmetic and geometric sequences and series, including those defined with recurrence relations, first terms, common differences or ratios, n^{th} terms, limits, or statements of convergence or divergence
   - Expanding binomials raised to a whole number power using the Binomial Theorem

10. Find limits of functions at specific values and at infinity numerically, algebraically, and graphically.
    - Applying limits in problems involving convergence and divergence

Geometry

11. Convert coordinates, equations, and complex numbers in Cartesian form to polar form and from polar form to Cartesian form.
    - Graphing simple polar equations in the polar coordinate plane
      Example: graphing $r = 2+2\cos\phi$ or $r = 2 + \sin 3\phi$
    - Graphing polar coordinates and complex numbers

Data Analysis and Probability

12. Determine the equation of a curve of best fit from a set of data by using exponential, quadratic, or logarithmic functions.